Stability of Proton and Maximally Symmetric Minimal Unification Model for Basic Forces and Building Blocks of Matter

Yue-Liang Wu

Institute of Theoretical Physics, Chinese Academy of sciences, Beijing 100080, China

(Dated: e-mail: ylwu@itp.ac.cn)

Abstract

With the hypothesis that all independent degrees of freedom of basic building blocks should be treated equally on the same footing and correlated by a possible maximal symmetry, we arrive at an 4-dimensional space-time unification model. In this model the basic building blocks are Majorana fermions in the spinor representation of 14-dimensional quantum space-time with a gauge symmetry $G_M^{4D} = SO(1,3) \times SU(32) \times U(1)_A \times SU(3)_F$. The model leads to new physics including mirror particles of the standard model. It enables us to issue some fundamental questions that include: why our living space-time is 4-dimensional, why parity is not conserved in our world, how is the stability of proton, what is the origin of CP violation and what can be the dark matter.

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The most important issues in elementary particle physics concern fundamental questions such as: what is the basic building blocks of nature? what is the basic symmetries of nature? what is the basic forces of nature? why our living space-time is 4-dimensional? why parity is not conserved in our world? how is the stability of proton? what can be the dark matter? what is origin of CP violation? why neutrinos are so light? In the standard model[1, 2, 3], quarks[4, 5] and leptons are regarded as the basic building blocks and described by the gauge symmetries $U(1)_Y \times SU(2)_L \times SU(3)_C$ and Lorentz symmetry SO(1,3). The gauge bosons corresponding to the symmetries mediate interactions among quarks and leptons via electromagnetic, weak and strong forces. The local Lorentz symmetry SO(1,3) reflects gravitational force.

The standard model has been tested by more and more precise experiments at the energy scale of order 100 GeV. The weak interaction is well described by the left-handed $SU(2)_L$ gauge symmetry based on the fact of parity (P) noninvariance[6]. The strong interaction described by the Yang-Mills gauge theory[7] with symmetry $SU(3)_C$ displays a behavior of asymptotic freedom[8, 9]. The standard model has been shown to be a renormalizable quantum field theory[10]. The strength of all forces has been turned out to run into the same magnitude at high energy scales[11], which makes it more attractive for the exploration of grand unification theories[12, 13, 14, 15]. SU(5) gauge model[13] is known to be a minimal grand unification theory. For massive neutrinos, SO(10) model[14, 15] may be regarded to be a minimal one. An interesting feature in the SO(10) model is that all the quarks and leptons in each family can be unified into a single spinor representation. One of the important predictions in either SU(5) or SO(10) model is proton decay. Namely proton is no longer a stable particle in the SU(5) and SO(10) models. As a consequence, the minimal SU(5) and SO(10) models have been strongly constrained by the current experimental data on proton decays.

In here we shall explore other possible unification models. For that, let us reanalyze what symmetry means in the grand unification models. We shall first examine the symmetries in the standard model. It is seen that symmetries are introduced to establish the relations among different quantum charges of quarks and leptons. The known quantum charges of quarks and leptons consist of isospin charges, color charges, lepton charges, spin charges and chiral-boost charges. Specifically, $SU(2)_L$ is introduced to describe the symmetry between two isospin charges, $SU(3)_C$ characterizes the symmetry among three color charges, SO(1,3)

reflects the symmetry among 2 spin charges and 2 chiral-boost charges. In the SO(10) model, SO(10) characterizes the symmetry of unified isospin-color-lepton charges. When treating all the quantum charges on the same footing[16], we arrive at a symmetry group $SO(1,3)\times SO(10)$, which may be regarded as a symmetry of 14-dimensional (14D) *quantum space-time*. This is because its dimensions are determined by the basic quantum charges of quarks and leptons (2 spin and 2 boost charges, 2 isospin charges, 3 color and 3 anticolor charges, lepton and antilepton charges). The independent degrees of freedom of quarks and leptons are given by the spinor representation of 14D quantum space-time. As there are 64 real independent degrees of freedom for each family quarks and leptons, the symmetry only based on the quantum charges of quarks and leptons cannot be a maximal symmetry that establishes possible correlations among the independent degrees of freedom of quarks and leptons. In other words, a large amount of independent degrees of freedom of quarks and leptons are not related via the symmetry group $SO(1,3) \times SO(10)$. It then becomes manifest that in the grand unification models one only considers symmetries among the basic quantum charges of building blocks rather than among all independent degrees of freedom of building blocks. In the present paper, we shall extend the usual grand unification models by considering a possible maximal symmetry among all independent degrees of freedom of basic building blocks.

As a principle in our present consideration, we make a simple hypothesis that *all independent degrees of freedom of basic building blocks should be treated equally on the same footing and correlated by a possible maximal symmetry in a minimal unified scheme.* For convenience of mention, we may refer such a hypothesis as a maximally symmetric minimal unification hypothesis (MSMU-hypothesis), and the resulting model as a maximally symmetric minimal unification model (MSMUM).

In order to establish possible correlations among all independent degrees of freedom of basic building blocks, we can infer from the above MSMU-hypothesis the following deduction: *fermions as basic building blocks of nature should be Majorana fermions in the spinor representation of high dimensional quantum space-time which is determined by the basic quantum charges of building blocks. The chirality of basic building blocks should be well-defined when parity is considered to be a good symmetry.* This deduction implies that the dimensions of quantum space-time should allow a spinor representation for both Majorana and Weyl fermions, which requires the quantum space-time to be at the dimensions D = 2

+ 4n ($n = 1, 2, \cdots$), i.e., D = 2, 6, 10, 14, 18, 22, 26, $\cdots$.

In the spirit of MSMU-hypothesis, the minimal dimension needed for a MSMUM is D=14. Thus the basic building blocks are Majorana fermions in the spinor representation of 14D quantum space-time. Namely each family of Majorana fermion has $128 = 2^7$ independent real degrees of freedom, which is twice to the quarks and leptons in the standard model. With the Majorana condition in the spinor representation of 14D quantum space-time, we will arrive at an interesting 4D space-time MSMUM with a gauge symmetry for each family $G_M^{4D} = SO(1,3) \times SU(32) \times U(1)_A$.

For an explicit demonstration, let us denote Ψ as fermionic building block in the spinor representation of 14D space-time. The Majorana condition implies that

$$\Psi = \Psi^{\hat{c}} = \hat{C}\bar{\Psi}^T \tag{1}$$

Here $\Psi^{\hat{c}}$ defines the charge conjugation in the 14D quantum space-time. The 128-dimensional spinor representation of Majorana fermion Ψ is found to have the form

$$\Psi = \begin{pmatrix} F_L + F'_R \\ F_R + F'_L \end{pmatrix} \tag{2}$$

with $F_{L,R}$ defined as

$$\begin{aligned} F_{L,R}^T &= [\ U_r, U_b, U_g, N, D_r, D_b, D_g, E, \\ &\quad D_r^c, D_b^c, D_g^c, E^c, -U_r^c, -U_b^c, -U_g^c, -N^c\ ]_{L,R}^T \\ F_{L,R}^{\prime T} &= [\ U'_r, U'_b, U'_g, N', D'_r, D'_b, D'_g, E', \\ &\quad D_r^{\prime c}, D_b^{\prime c}, D_g^{\prime c}, E^{\prime c}, -U_r^{\prime c}, -U_b^{\prime c}, -U_g^{\prime c}, -N^{\prime c}\ ]_{L,R}^T \end{aligned} \tag{3}$$

and the charge conjugation matrix is given by

$$\hat{C} = i\sigma_1 \otimes \sigma_2 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes \sigma_2 \tag{4}$$

which satisfies $\hat{C}^\dagger = \hat{C}^{-1} = -\hat{C}^T = -\hat{C}$ and $\hat{C}\hat{C}^\dagger = 1$. All the fermions $\psi = U_i,\ D_i,\ E,\ N, \cdots$ are four complex component fermions defined in the 4D. The indices "L" and "R" denote the left-handed and right-handed fermions in 4D, i.e., $\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi$. The index "c" represents the charge conjugation in 4D, $\psi^c = C\bar{\psi}^T$ with $C = i\gamma_0\gamma_2 = i\sigma_3 \otimes \sigma_2$.

For convenience of discussions, we present the explicit form of the gamma matrices $\hat{\Gamma}_{\hat{I}} = (\gamma_a, \Gamma_I)$ in the spinor representation of 14D quantum space-time

$$
\begin{aligned}
\gamma_0 &= 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_1 \otimes 1 \ , \\
\gamma_1 &= i\ 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_1 \ , \\
\gamma_2 &= i\ 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_2 \ , \\
\gamma_3 &= i\ 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \ , \\
\Gamma_1 &= \sigma_1 \otimes \sigma_1 \otimes 1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \otimes 1 \ , \\
\Gamma_2 &= \sigma_1 \otimes \sigma_2 \otimes 1 \otimes \sigma_3 \otimes \sigma_2 \otimes \sigma_3 \otimes 1 \ , \\
\Gamma_3 &= \sigma_1 \otimes \sigma_1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \otimes \sigma_3 \otimes 1 \ , \\
\Gamma_4 &= \sigma_1 \otimes \sigma_2 \otimes 1 \otimes \sigma_2 \otimes 1 \otimes \sigma_3 \otimes 1 \ , \\
\Gamma_5 &= \sigma_1 \otimes \sigma_1 \otimes 1 \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_3 \otimes 1 \ , \\
\Gamma_6 &= \sigma_1 \otimes \sigma_2 \otimes 1 \otimes \sigma_1 \otimes \sigma_2 \otimes \sigma_3 \otimes 1 \ , \\
\Gamma_7 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \ , \\
\Gamma_8 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_2 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \ , \\
\Gamma_9 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \ , \\
\Gamma_{10} &= \sigma_2 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1
\end{aligned}
\tag{5}
$$

where σ_i $(i = 1, 2, 3)$ are Pauli matrices, and "1" is understood as a 2×2 unit matrix. We may define two gamma matrices from $\hat{\Gamma}_{\hat{I}}$ $(\hat{I} = 0, \cdots, 13)$ as

$$
\begin{aligned}
\Gamma_{15} &= \hat{\Gamma}_0 \cdots \hat{\Gamma}_{13} = \sigma_3 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3 \otimes 1 \\
\Gamma_{11} &= \Gamma_1 \cdots \Gamma_{10} = \sigma_3 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1
\end{aligned}
\tag{6}
$$

with $\Gamma_{15}\hat{\Gamma}_{\hat{I}} = -\hat{\Gamma}_{\hat{I}}\Gamma_{15}$ and $\Gamma_{11}\Gamma_I = -\Gamma_I\Gamma_{11}$. The Weyl representation of Ψ is defined through the projector operators $P_{W,E} = (1 \mp \Gamma_{15})/2$ with $P^2_{W,E} = P_{W,E}$

$$
\begin{aligned}
\Psi_W &= P_W\Psi = \frac{1}{2}(1 - \Gamma_{15})\Psi \equiv \begin{pmatrix} F_L \\ F_R \end{pmatrix} , \\
\Psi_E &= P_E\Psi = \frac{1}{2}(1 + \Gamma_{15})\Psi \equiv \begin{pmatrix} F'_R \\ F'_L \end{pmatrix}
\end{aligned}
\tag{7}
$$

Here Ψ_W and Ψ_E may be mentioned as 'westward' and 'eastward' fermions in order to

distinguish with the left- and right- handed fermions defined via the projector operators $P_{L,R} = (1 \mp \gamma_5)/2$ in 4D space-time.

The westward fermion Ψ_W is a Majorana-Weyl fermion, it contains 64 independent degrees of freedom that exactly represent quarks and leptons in each family with right-handed neutrino. The eastward Majorana-Weyl fermion Ψ_E may be regarded as mirror particles and mentioned as mirroquarks and mirroleptons.

We can construct following tensors by Γ-matrices

$$
\begin{aligned}
&T^U \equiv (\Sigma^{IJ},\ \Gamma_{11}\Sigma^{IJKL},\ \Gamma_{11}) \\
&T_5^V \equiv (\Sigma^{IJK},\ i\Gamma_{11}\Sigma^{IJK}),\ T^{\hat{V}} \equiv (T^U,\ i\Gamma_{15}T_5^V) \\
&T_5^W \equiv (\Sigma^I,\ i\Gamma_{11}\Sigma^I,\ \Sigma^{IJKLM}),\ T_5^{\hat{W}} \equiv (T_5^W, \Gamma_{15}T^U)
\end{aligned}
\tag{8}
$$

with $\Sigma^I \equiv \frac{1}{2}\Gamma^I$, $\Sigma^{IJ} = \frac{i}{4}[\Gamma^I, \Gamma^J]$ and others are the higher order antisymmetric tensors. Under charge conjugation and parity transformation, we have

$$
\begin{aligned}
&\hat{C}T^{\hat{V}}\hat{C}^\dagger = -(T^{\hat{V}})^T, \quad \Gamma^0 T^{\hat{V}}\Gamma^0 = (T^{\hat{V}})^\dagger \\
&\hat{C}T_5^{\hat{W}}\hat{C}^\dagger = (T_5^{\hat{W}})^T, \quad \Gamma^0 T_5^{\hat{W}}\Gamma^0 = -(T_5^{\hat{W}})^\dagger \\
&\hat{C}\Sigma^{ab}\hat{C}^\dagger = -(\Sigma^{ab})^T, \quad \Gamma^0\Sigma^{ab}\Gamma^0 = (\Sigma^{ab})^\dagger
\end{aligned}
\tag{9}
$$

where $\Sigma^{ab} = \frac{1}{4i}[\gamma^a, \gamma^a]$ $(a, b = 0,\ 1,\ 2,\ 3)$ are the generators of Lorentz group SO(1,3). Here the matrices

$$
T^{\hat{A}} = (T^{\hat{V}}, T_5^{\hat{W}})\ , \quad (\hat{A} = 1, \cdots, 1024)
\tag{10}
$$

form the generators of symmetry group $SU(32) \times U(1)_A$. Where $T^{\hat{V}}$ $(\hat{V} = 1, \cdots, 496)$ form the generators of subgroup $SO(32)$, T^U $(U = 1, \cdots, 256)$ the generators of a subgroup $SU(16) \times U(1)$ and Σ^{IJ} the generators of a subgroup SO(10).

It is of interest to observe that although the 128-dimensional spinor representation of Majorana fermion Ψ is defined in the 14D quantum space-time, while its motion cannot realize in the whole 14D space-time corresponding to the 14D quantum space-time. This is because no kinetic term can exist in the corresponding 10D space, and its motion can only be emergent in an 4D space-time out of the 14D space-time. This can be seen from the identities

$$
\begin{aligned}
&\bar{\Psi}\Gamma^I i\partial_I\Psi \equiv \frac{1}{2}\partial_I\left(\bar{\Psi}i\Gamma^I\Psi\right) \\
&\bar{\Psi}\gamma^a i\partial_a\Psi \equiv \frac{1}{2}[\bar{\Psi}\gamma^a i\partial_a\Psi - i\partial_a(\bar{\Psi})\gamma^a\Psi]
\end{aligned}
\tag{11}
$$

with $a = 0, 1, 2, 3$, and $I = 1, \cdots, 10$. Here the Majorana condition has been used. As there exists no motion in the 10D space, the 32-dimensional spinor representation of 10D quantum space is found to have a maximal symmetry $SU(32) \times U(1)_A$ in the kinetic 4D space-time.

The number of families remains a puzzle. When treating the observed three families equally on the same footing, a maximal family symmetry among them is $SU(3)_F$ with the chiral-valued generators

$$T^u = (\lambda^i,\ \gamma_5\lambda^s), \quad (\lambda^i)^T = -\lambda^i, \quad (\lambda^s)^T = \lambda^s \tag{12}$$

and $(i = 1, 2, 3;\ s = 1, \cdots, 5)$. Where λ^i form the generators of $SO(3)_F \in SU(3)_F$.

We are now in the position to write down the Lagrangian of MSMUM in the kinetic 4D space-time for the fermionic building blocks

$$\mathcal{L}_F^{4D} \ = \ \frac{1}{2}\bar{\Psi}\gamma^a iD_a\Psi \equiv \frac{1}{2}\bar{\Psi}\hat{\gamma}^a E_a^\mu iD_\mu\Psi \tag{13}$$

which is self-Hermician. Here E_a^μ is introduced as frame fields. The covariant derivative D_μ is defined as

$$D_\mu = \partial_\mu - i\omega_\mu^{ab}\Sigma_{ab} - ig_U\Omega_\mu^{\hat{A}}T^{\hat{A}} - ig_F F_\mu^u T^u \tag{14}$$

The above Lagrangian has a maximal gauge symmetry

$$G_M^{4D} = SO(1,3) \times SU(32) \times U(1)_A \times SU(3)_F \tag{15}$$

where the three families $\Psi = (\Psi_1,\ \Psi_2,\ \Psi_3)$ belong to the fundamental representation of $SU(3)_F$.

A minimal scalar field that couples to fermions is in the representations $\Phi_F = \gamma_0\lambda^s\Phi_s^{\hat{W}}T_5^{\hat{W}} \equiv \gamma_0\lambda^s(\gamma_5\Sigma_s^{\hat{W}} + i\Pi_s^{\hat{W}})T_5^{\hat{W}}$ $(s = 0, 1, \cdots, 5;\ \hat{W} = 1, \cdots, 527)$. The self-Hermician Lagrangian for Yukawa interactions is

$$\mathcal{L}_Y^{4D} \ = \ \frac{1}{2}g_Y\Psi^\dagger\Phi_F\Psi = \frac{1}{2}g_Y\bar{\Psi}\lambda^s\Phi_s^{\hat{W}}T_5^{\hat{W}}\Psi \tag{16}$$

Here the minimal scalar field Φ_F contains representations $(272)_{W,E} = (10+\bar{1}0+126+1\bar{2}6)_{W,E}$ that are needed for generating masses of quarks and leptons as well as their mirroparticles via spontaneous symmetry breaking.

A scalar field that interacts with gauge bosons can be different from the one that couples to fermions. Such that a scalar field in the adjoint representation of SU(32) $\hat{\Phi} \equiv \hat{\Phi}^{\hat{A}}T^{\hat{A}}$ only

couples to gauge bosons. The self-Hermician Lagrangian for minimal scalar fields is given by

$$\mathcal{L}_S^{4D} = TrD_\mu \Phi_F^\dagger D_\mu \Phi_F + TrD_\mu \hat{\Phi}^\dagger D_\mu \hat{\Phi} \tag{17}$$

In general, a total self-Hermician Lagrangian is

$$\mathcal{L}^{4D} = \mathcal{L}_F^{4D} + \mathcal{L}_G^{4D} + \mathcal{L}_S^{4D} + \mathcal{L}_Y^{4D} + \mathcal{L}_H^{4D} \tag{18}$$

Here $\mathcal{L}_G^{4D}$ represents the Yang-Mills gauge interactions and $\mathcal{L}_H^{4D}(\hat{\Phi}, \Phi_F)$ the Higgs potential. Note that when taking $g_U = g_F = g_Y \equiv g_o$, we arrive at an MSMUM with a single coupling constant g_o.

Here we are able to issue some fundamental questions:

1) as quarks and leptons belong to the Majorana-Weyl representation of 14D quantum space-time, their motion has been shown to be emergent only in the 4D space-time. This naturally answers two important issues: why our living space-time that consists of quarks and leptons is 4-dimensional, and why parity becomes no invariance in our world that is made of quarks and leptons;

2) fermionic building blocks in the MSMUM are twice as those in the standard model. The additional building blocks are regarded as mirroquarks and mirroleptons, which can be seen more explicitly from the symmetry decomposition $SU(16)_W \times SU(16)_E \times U(1) \in SU(32)$. Of particular, when the mirroparticles form stable states that only weakly interact with the ordinary matter, the corresponding mirromatter can become interesting candidate of dark matter observed in our universe;

3) unlike to the usual grand unification theories, proton may become rather stable in the MSMUM. This is because the subgroup $SU(16)_W \in SU(32)$ provides a maximal gauge symmetry among quarks and leptons[17], so that the gauge interactions for all quarks and leptons are associated with different gauge bosons. When $SU(16)_W$ is appropriately broken down to the symmetries in the standard model without causing, in the mass eigenstates, a mixing among gauge bosons that can mediate proton decays, then proton remains stable. Namely the stability of proton in the MSMUM relies on whether a mixing occurs among relevant gauge bosons in the mass eigenstates;

4) when the constraints from proton stability become weak, some symmetry breaking scales can be low in the MSMUM. It is specially interesting to look for new particles at TeV scales.

But possible intermediate energy scales and symmetry breaking scenarios should match to the constraints from the running coupling constants;
5) as the Majorana condition in the MSMUM leads to a self-Hermician Lagrangian, it then implies that CP symmetry should be broken down spontaneously[18, 19, 20];
6) since the symmetry is maximal in the MSMUM with a minimal parameter, it is expected to be more predictive. Especially, the minimal scalar field Φ_F in the Yukawa coupling contains $(10 + \bar{1}0 + 126 + 1\bar{2}6)$ representations needed for mass generation and see-saw mechanism, which helps to understand from vacuum structures of symmetry breaking pattern how quarks and leptons get masses and mixing, and why neutrinos are so light;
7) like grand unification models, symmetry breaking scenarios and vacuum structures will be the most important issues in the MSMUM. A simple symmetry breaking scenario can be: $SU(32) \rightarrow SU(16)_W \times SU(16)_E \times U(1)$
$SU(16)_W \rightarrow SU(8)_L \times SU(8)_R \rightarrow SU(4)_L \times SU(2)_L \times SU(4)_R \times SU(2)_R \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_{em}$. The properties of mirror particles will depend on the symmetry breaking patterns of $SU(16)_E$. In general, for breaking a maximal symmetry to a symmetry of real world, it is crucial to apply for suitable symmetry breaking mechanisms, like the dynamically spontaneous symmetry breaking realized in QCD[21];
8) when extending the principle of MSMU-hypothesis to the space-time symmetry, we shall arrive at supersymmetric MSMUM.

Last but not least, we would like to address that the above resulting 4-dimensional MSMUM naturally match to the so-called no-go theorem proved by Coleman and Mendula[22]. The main assumption made here is that the Majorana fermions belonging to the spinor representation of 14D quantum space-time are equally treated on the same footing and directly identified to the basic building blocks of quarks and leptons. In general, there are many possibilities for imposing different Majorana spinor structures. For different spinor structures, one then arrives at different geometries of high dimensional space-time, which leads to different physics. What we have demonstrated in this note is that when choosing the Majorana spinor structure in the 14D space-time to be directly identified to the quarks and leptons, it then automatically leads to a unique solution with kinetic term only appearing in the 4D space-time for each generation quarks and leptons, the remaining 10D space becomes an internal space without motions. This may be regarded as an alternative dimension reduction via choosing the Majorana spinor structure based on the observations

in the real world of elementary particles. In general, without requiring a specific Majorana spinor structure in a high dimensional space-time to be assigned to the observed quarks and leptons, the Majorana spinor fermions can in principle have motions in the whole high dimensional space-time, namely there should be no constraints for the dimensions of motion in the most general case. One may compare such a reduction of dimension of motion via specifying a spinor structure with the compactification approach in which the high dimensions are compacted to lead to a 4D space-time. For different compactifications, one yields different spinor structures. The well-known example is the string theory, where the Majorana spinors belonging the representations in the 10D space-time are not required to directly relate to the quarks and leptons, so that they can have motions in the whole 10D space-time. For such a case, one needs to make an appropriate compactification to yield an effective 4D space-time theory. There exist in general many patterns for the compactifications in the string theories. Obviously, different compactifications lead to different spinor structures which are corresponding to different physics.

In conclusion, starting from a simple hypothesis, we are led to a 4-dimensional MSMUM with the gauge symmetry $G_M^{4D} = SO(1,3) \times SU(32) \times U(1)_A \times SU(3)_F$.

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